$\therefore\ d(a/x)=2k(n+1)d\delta.$

$\therefore\ dp=2k(n+1)ag\delta d\delta,\ p=agk(n+1)\delta^2.$

Let l=height of a homogenous atmosphere of density ρ exerting a pressure p_0.

$\therefore\ p_0=g\rho l.$

$$\therefore\ \frac{p}{p_0}=(n+1)\ \frac{a}{l}.\frac{k\delta^2}{\rho},\ \text{when } p=p_0,\ \delta=\rho.$$

$$\therefore\ 1=(n+1)\ \frac{a}{l}.k\rho,\ \text{or } n=\frac{l/a}{k\rho}-1.$$

a=6366738 meters, $k\rho$=.000147192.

l=.76 (as many times as mercury is as heavy as air)=7993.15 meters.

$\therefore\ l/a$=.00125545.

$$\therefore\ n=\frac{.00125545}{.000147192}-1=7.5,\ \text{a value of } n \text{ too large.}$$

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

146. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

If the driving-wheels of Locomotive No. 200 on the Pennsylvania Railroad, m=7 feet in diameter, turn n=20 times in p=3 seconds, and lose r=12% of their forward motion by slipping on the smooth steel rails, at what rate per hour is the locomotive moving over the rails?

Solution by W. P. WEBBER, Mississippi Normal College, Houston, Miss.

$m\pi$=circumference of wheel, $mn\pi$=number of times the circumference is applied to the rail, and is therefore the distance the engine travels without slipping in p seconds.

Hence, $\dfrac{mn\pi}{p}$=distance engine travels in 1 second without slipping, and $\dfrac{mn(100-r)\pi}{100p}$=actual distance engine travels in 1 second, since it slips back r% of the distance it travels.

Substituting numbers for the letters, we have for the distance the engine travels in 1 second, $\dfrac{7\times20(100-12)\pi}{100\times3}$feet.

Hence, the rate of the engine in miles per hour is

$$\frac{7\times20\times88\times\pi\times3600}{100\times3\times5280}=28\pi \text{ miles.}$$

ALGEBRA.

123. Proposed by ELMER SCHUYLER, B. Sc., Professor of German and Mathematics, Boys' High School, Reading, Pa.

$$\left(\frac{1+x}{1-x}\right)^{\frac{1}{4}}+\sqrt{\frac{1-a}{1+a}}\sqrt[4]{\frac{1-x}{1+x}}=2\sqrt[4]{\frac{1-a^2}{(1+a)^2}}, \text{ and } \sqrt{a^2-x^2}+x\sqrt{a^2-1}=a^2\sqrt{1-x^2}.$$

[*Haddon.*]

Solution by JOHN A. VAN GROOS, Fellow of Mathematics, University of Oregon, Eugene; Ore.; R.L.MOORE, Student in University of Texas, 2206 San Marcos Street, Austin, Tex,; and G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

(1). Multiply $\left(\frac{1+x}{1-x}\right)^{\frac{1}{4}}+\sqrt{\frac{1-a}{1+a}}\sqrt[4]{\frac{1-x}{1+x}}=2\sqrt[4]{\frac{1-a^2}{(1+a)^2}}=2\sqrt[4]{\frac{1-a}{1+a}}$ through by

$$\left(\frac{1+x}{1-x}\right)^{\frac{1}{4}}. \quad \therefore\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}-2\left(\frac{1-a}{1+a}\right)^{\frac{1}{4}}\left(\frac{1+x}{1-x}\right)^{\frac{1}{4}}+\left(\frac{1-a}{1+a}\right)^{\frac{1}{2}}=0.$$

$$\therefore\left[\left(\frac{1+x}{1-x}\right)^{\frac{1}{4}}-\left(\frac{1-a}{1+a}\right)\right]^2=0. \quad \therefore \frac{1+x}{1-x}=\frac{1-a}{1+a}.$$

$\therefore x=-a.$

(2). $\sqrt{(a^2-x^2)}=a^2\sqrt{(1-x^2)}-x\sqrt{(a^2-1)}.$

$$a^2-x^2=a^4-a^4x^2-2a^2x\sqrt{[(1-x^2)(a^2-1)]}+a^2x^2-x^2.$$

$\therefore\ 2a^2x\sqrt{[(1-x^2)(a^2-1)]}=a^2(1-x^2)(a^2-1).$

$\therefore\ \{2x-\sqrt{[(1-x^2)(a^2-1)]}\}\sqrt{[(1-x^2)(a^2-1)]}=0.$

$\therefore\ 4x^2=(1-x^2)(a^2-1)$ or $x^2=1.$

$\therefore\ x=-1,\ +1,$ or $\pm\sqrt{\frac{a^2-1}{3+a^2}}.$

The values $x=-1$ and $\sqrt{\frac{a^2-1}{3+a^2}}$ are the values satisfying the equation as given.

124. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A certain quantity of alcohol diluted with water so that in one liter there are c liters of pure alcohol, is mixed n times successively with p times the quantity of alcohol diluted so that 1 liter contains a liter of pure alcohol. How much pure alcohol does one liter of the nth mixture contain ?